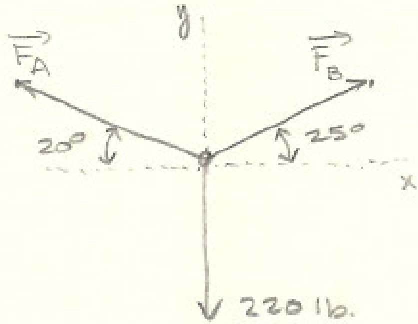
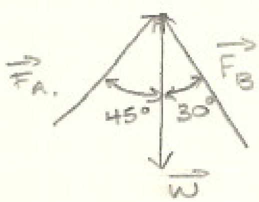
EX. 3-1

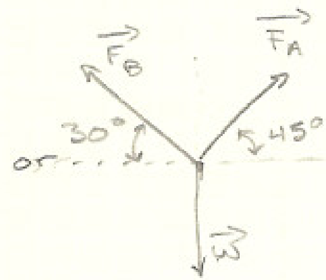


SOLUTION:

$$\vec{F_A} = F_A(-\cos 20° \vec{i} + \sin 20° \vec{j})$$
$$\vec{F_B} = F_B(\cos 25° \vec{i} + \sin 25° \vec{j})$$
$$\vec{W} = 220(-\vec{j}) = -220\vec{j}$$
$$\vec{R_x} = 0 \qquad \vec{R_y} = 0$$
$$R_x = -F_A\cos 20° \vec{i} + F_B \cos 25° \vec{i} = 0$$
$$R_y = F_A \sin 20° \vec{j} + F_B \sin 25° \vec{j} - 220\vec{j} = 0$$
$$\therefore \quad F_A = 282 \text{ lb.}$$
$$F_B = 292 \text{ lb.}$$

EX. 3-2.



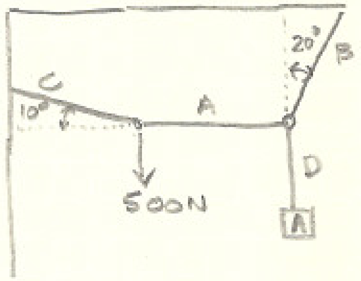


SOLUTION: $R_x = 0 \quad R_y = 0$

$$W = 50 \text{ kg} \cdot 9.81 \text{ m/s}^2 = 490.5 \text{ N.}$$

$$\vec{F_A} = F_A(\cos 45° \vec{i} + \sin 45° \vec{j})$$
$$\vec{F_B} = F_B(-\cos 30° \vec{i} + \sin 30° \vec{j})$$
$$W = 490.5(-\vec{j})$$
$$R_x = F_A \cos 45° \vec{i} - F_B \cos 30° \vec{i} = 0$$
$$R_y = F_A \sin 45° \vec{j} + F_B \sin 30° \vec{j} - 490.5\vec{j} = 0$$
$$\therefore \quad F_A = 254 \text{ N} \qquad F_B = 359 \text{ N.}$$

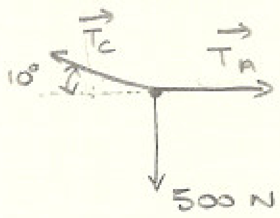
EX. 3-6.



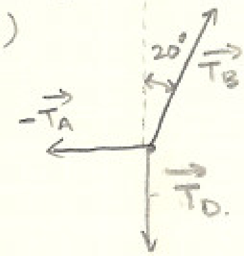
FIND THE TENSION IN EACH CABLE & THE MASS OF A.

SOLUTION: DRAW 3 FBD.

i)



ii)



iii)

i) $R_x = 0 \quad R_y = 0$

$W = -500\vec{j}$

$\vec{T_A} = T_A\vec{i}$

$\vec{T_C} = T_C(-\cos 10°\vec{i} + \sin 10°\vec{j})$

$R_x = 0 = T_A\vec{i} - T_C\cos 10°\vec{i}$

$R_y = 0 = T_C\sin 10°\vec{j} - 500\vec{j}$

$\therefore T_C = 2879\text{N}$

$T_A = 2835\text{N}$.

ii) $R_x = 0 \quad R_y = 0$

$\vec{T_A} = -2835\vec{i}$

$\vec{T_B} = T_B(\cos 70°\vec{i} + \sin 70°\vec{j})$

$\vec{T_D} = -T_D\vec{j}$

$R_x = 0 = -2835\vec{i} + T_B\cos 70°\vec{i}$

$R_y = 0 = T_B\sin 70°\vec{j} - T_D\vec{j}$

$\therefore T_B = 8289\text{ N}$

$T_D = 7789\text{N}$.

iii) $R_x = 0 \quad R_y = 0$

$$R_y = 0 = -7789\vec{j} + W\vec{j}$$

$$\therefore \quad W = 7789\,N$$

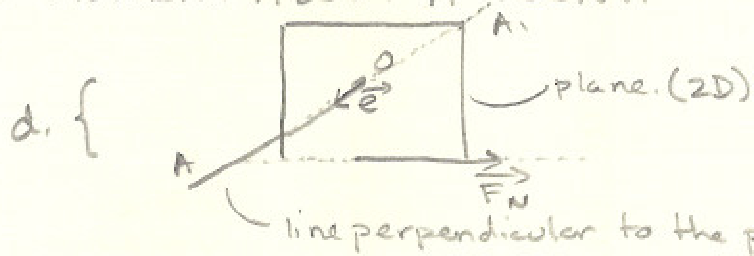
$$\text{mass of } A = \frac{7789\,N}{9.81\,m/s^2} = 734\text{ kg.}$$

CHAPTER 4. RIGID BODIES, EQUIVALENT TO FORCE & MOMENT.

§4.1 INTRODUCTION.

- RIGID BODIES
  - CONCURRENT RESULTS IN SINGLE FOR.
  - COPLANAR, RESULTS IN A SINGLE FOR AND A MOMENT.

§4.2 MOMENT ABOUT A POINT.

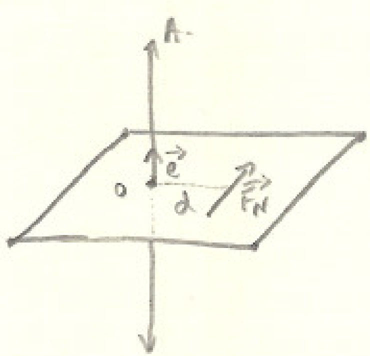


$$\boxed{\vec{M_o} = F_N\, d\, \vec{e}}$$

O: is the moment center

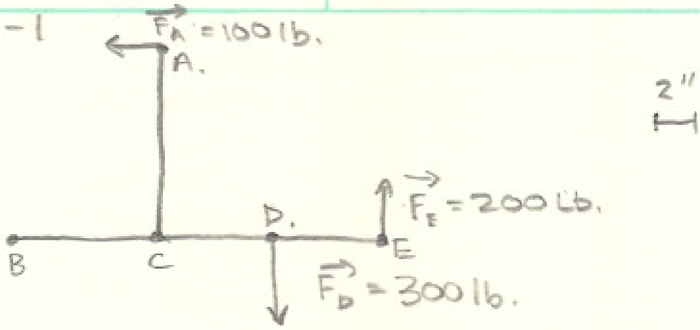
d: is the distance between the force applied and the point (AKA - moment arm).

AA: The axis of the moment.



MOMENT IS EXPRESSED $N \cdot m$ or $lb \cdot ft$.

EX. 4-1



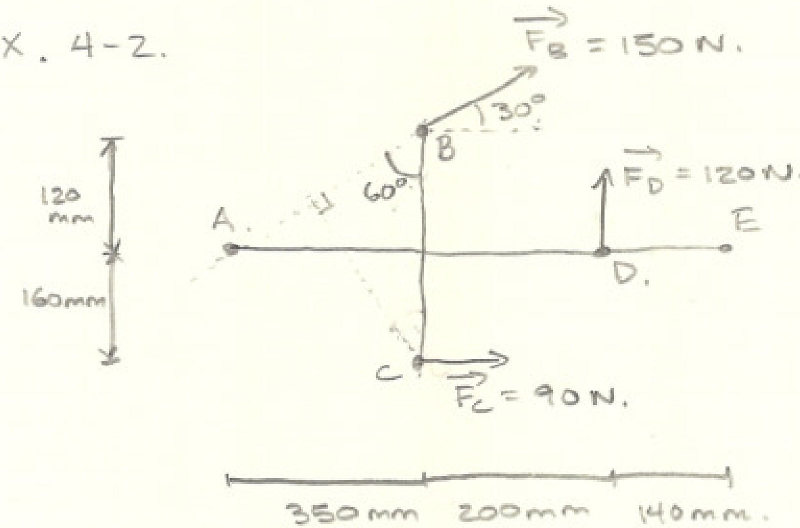
SOLUTIONS.

$$M_E(\vec{F_A}) = 100\text{ lb} \cdot 10'' = 1000 \text{ lb}\cdot\text{in}$$

$$M_A(\vec{F_E}) = 200\text{ lb} \cdot 1\text{ ft} = 200 \text{ lb}\cdot\text{ft}.$$

$$M_B(\vec{F_D}) = 300\text{ lb} \cdot 14\text{ in} = 4200 \text{ lb}\cdot\text{in}.$$

EX. 4-2.



SOLUTIONS:

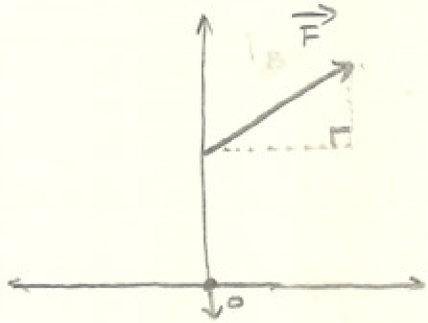
$$M_B(\vec{F_C}) = 90 \cdot (0.16 + 0.12) = 25.2 \text{ N}\cdot\text{m}.$$

$$M_A(\vec{F_D}) = 120 \cdot (0.35 + 0.2) = 66.0 \text{ Nm}$$

$$M_C(\vec{F_B}) = 150 \cdot ((0.16 + 0.12)\sin 60°) = 36.4 \text{ Nm}.$$

§ 4.2.1 PRINCIPLES OF MOMENTS.

THE MOMENT $\vec{M}$ OF THE RESULTANT $\vec{R}$ OF A SECTION OF FORCES w.r.t. ANY POINT OR AXIS IS EQUAL TO THE VECTOR SUM OF THE MOMENTS OF INDIVIDUAL FORCES OF THE SYSTEM w.r.t. THE SAME PT. OR AXIS.



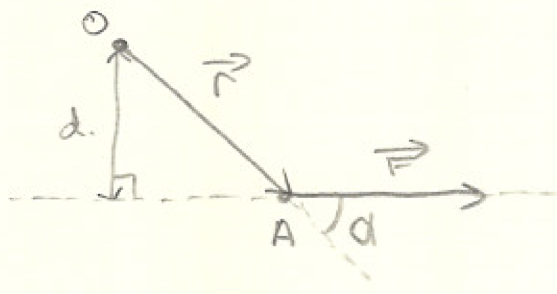
$$\overrightarrow{M_o}(\vec{F}) = \overrightarrow{M_o}(F_x\, i) + \overrightarrow{M_o}(F_y\, \vec{j})$$

§ 4.3 VECTOR REPRESENTATION OF A MOMENT.

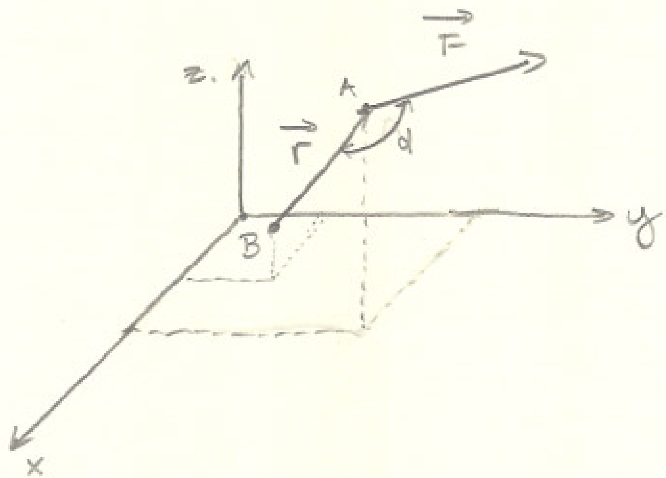
THE MOMENT OF A FORCE $\vec{F}$ ABOUT A POINT O.

$$\vec{M_o} = \vec{r} \times \vec{F}$$

$\vec{r}$: a position vector from point O to point A on the line of action.



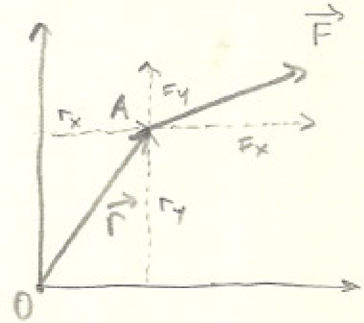
2D.



$\vec{r_{A/B}} \cdot F$

3D.

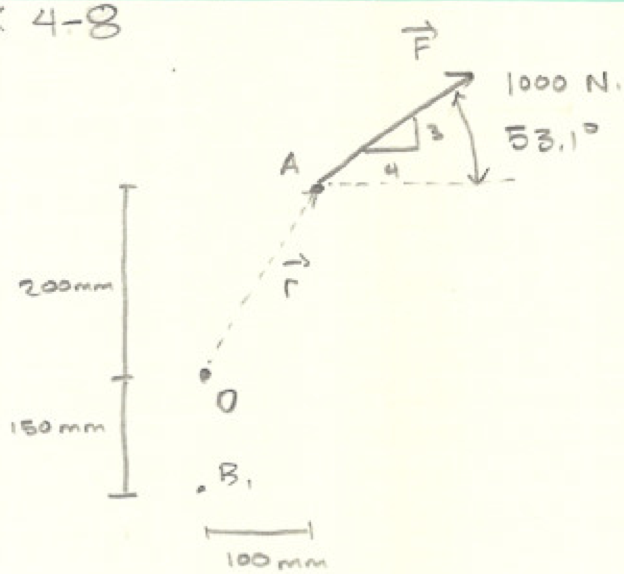
EX.



$$\vec{M_o} = \vec{r} \times \vec{F} = (r_x F_y - r_y F_x)k$$

$$M_o = r_x F_y - r_y F_x$$

EX 4-8



$\vec{F} = 1000\left(\cos(53.1)\vec{i} + \sin(53.1)\vec{j}\right)$

$\vec{F} = 800\vec{i} + 600\vec{j}$

$\vec{r} = 0.1\vec{i} + 0.2\vec{j}$

$\vec{M} = (r_x F_y - r_y F_x)\vec{k}$

$= ((0.1)(600) - (0.2)(800))\vec{k}$

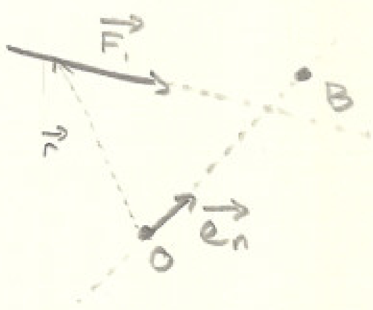
$= -100 \text{ N}\cdot\text{m}\ \vec{k}$

means that $\vec{e}$ is negative. ∴ CW rotation of the moment.

$\vec{M} = 100 \text{ N}\cdot\text{m}$ ↻

§ 4-3.2 MOMENT OF A FORCE ABOUT A LINE.

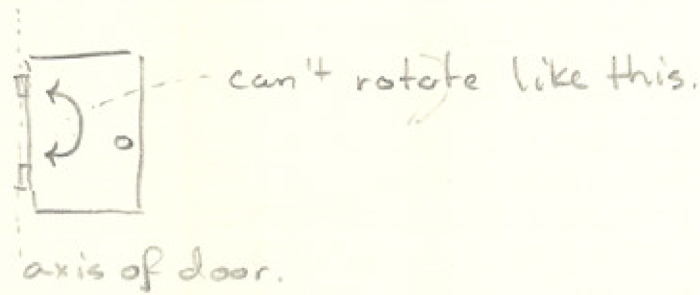
GIVEN A LINE OB AND A UNIT VECTOR, $\vec{e_n}$ ALONG THE LINE OB.



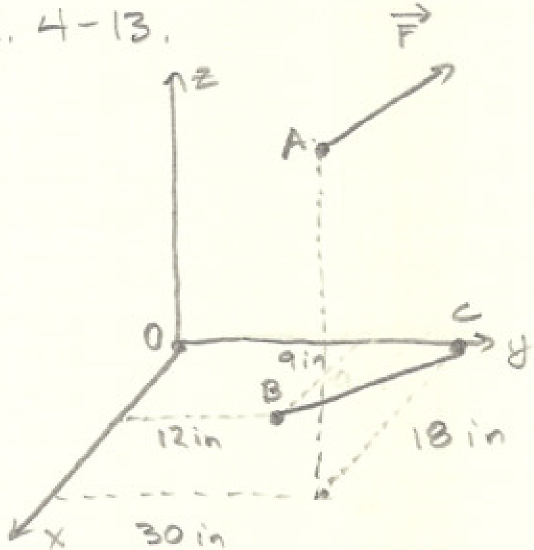
$\overrightarrow{M_{OB}} = (\vec{M_O} \cdot \vec{e_n})\vec{e_n}$

$\overrightarrow{M_{OB}} = (\vec{e_n} \cdot (\vec{F} \times \vec{r}))\vec{e_n}$

Note that is a force is on the same plane as the axes of a moment then there is no moment.



EX. 4-13.



$$\vec{F} = 60\vec{i} + 100\vec{j} + 120\vec{k}$$

SOLUTION:

$B(9,12,0) \quad C(0,30,0)$

$$\vec{r}_{C/B} = \vec{r}_C - \vec{r}_B$$
$$= (0\vec{i} + 30\vec{j} + 0\vec{k}) - (9\vec{i} + 12\vec{j} + 0\vec{k})$$
$$= -9\vec{i} + 42\vec{j}$$

now we find the unit vector on that line.

$$\vec{e}_{BC} = \frac{\vec{r}_{C/B}}{r_{C/B}} = \frac{-9\vec{i} + 18\vec{j}}{\sqrt{(-9)^2 + (18)^2}} = -0.4472\vec{i} + 0.8944\vec{j}$$

$$=$$

$\overrightarrow{M_C} = \overrightarrow{r_{A/C}} \times \vec{F}$

$\overrightarrow{r_{A/C}} = \overrightarrow{r_A} - \overrightarrow{r_C} = (18\vec{i} + 30\vec{j} + 32\vec{k}) - (30\vec{j})$

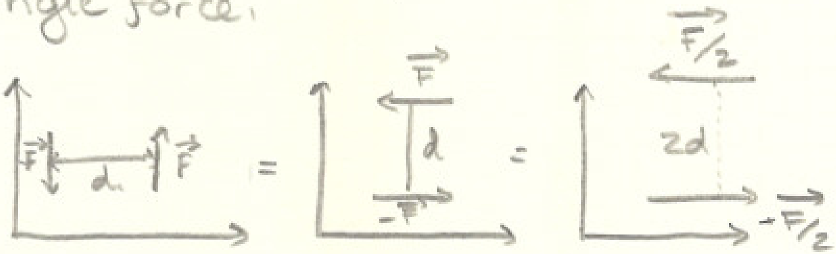
$= 18\vec{i} + 32\vec{k}$

$\overrightarrow{M_C} = (18\vec{i} + 32\vec{k}) \times (60\vec{i} + 100\vec{j} + 120\vec{k})$

$= -3200\vec{i} - 240\vec{j} + 1800\vec{k}$

§ 4-4 COUPLES.

the moment of a couple about any point B the same, A couple can be simplified to a single force.

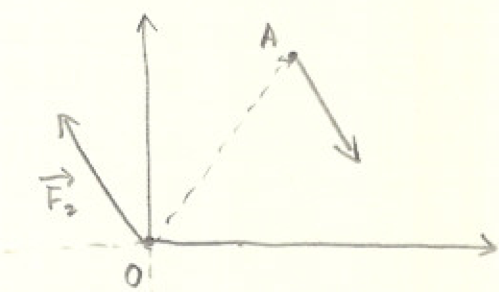


$M = Fd.$

moment of the couple

Any point will give the same moment.

EX. 4-16



Since we can pick any pt. to call our center we opt to use O. b/c it simplifies the math.

Solution: $\vec{M} = \overrightarrow{M_O} + \overrightarrow{M_{O_2}}$

$= dF$

$= (2\vec{i} + 2\vec{j}) \times (100\vec{i} - 50\vec{j})$

$= -300\vec{k}$

AMPAD

§ 4.6 SIMPLIFICATION OF A FORCE SYSTEM.

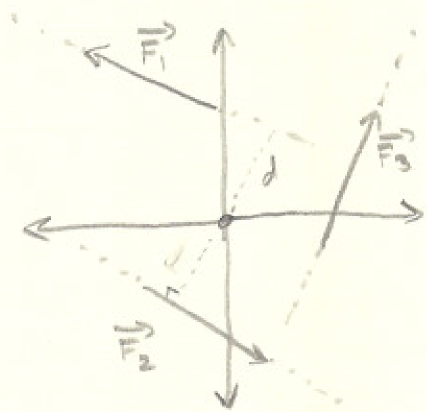
Two force systems are eqivilant if they produce the same external effect when applied to a rigid body, they have the same resultant and moment about the same point.

A force system → a resultant. { concurrent, coplanar, parallel

A force system → a couple { parallel, coplanar

A force system → a resultand & couple { 3D.

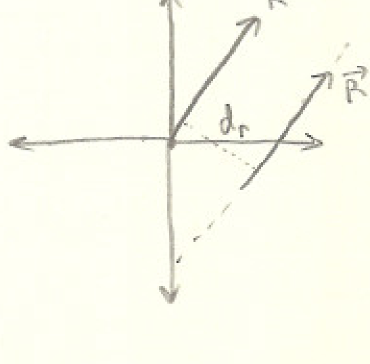
EX. COPLANAR FORCE SYSTEM



RESULTANT: $\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = \Sigma\vec{F}$

MAGNETUDE: $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

DIRECTION (Line of Action): CALC θ AS NORMAL. but b/c $\vec{R}$ & $\Sigma\vec{F}$ must also hande the same moment.



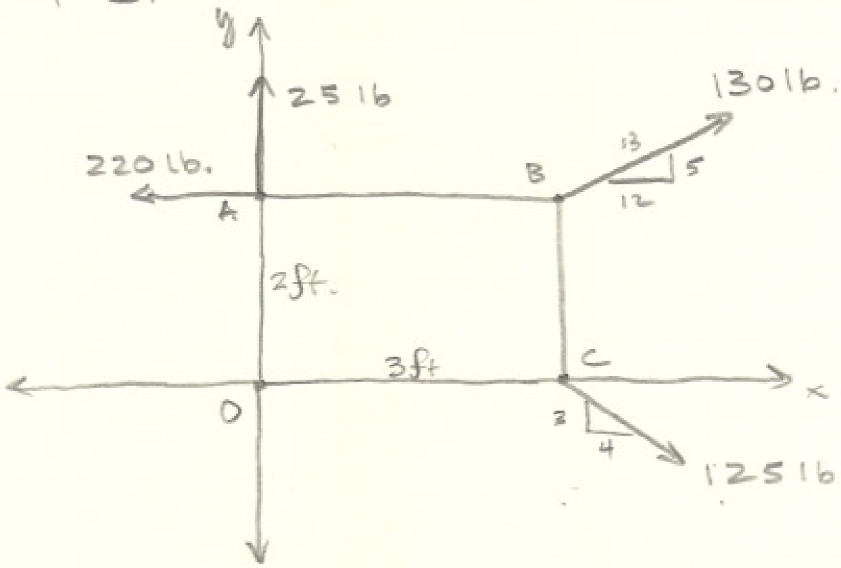
to find the line of action.

$R d_r = F_1 d_1 + F_2 d_2 + F_3 d_3 = \Sigma F_i d_i$

$d_r = \frac{\Sigma F_i d_i}{R} = \frac{\Sigma F_i d_i}{\Sigma F}$

If $R=0$ then we have a couple. There is no result vector and just moments.

EX. 4-21



Solution:

$$R_x = \Sigma F_x = -220 + 130\left(\frac{12}{13}\right) + 125\left(\frac{4}{5}\right)$$

$$R_x = \Sigma F_x = 0$$

$$R_y = \Sigma F_y = 25 + 130\left(\frac{5}{13}\right) + 125\left(\frac{3}{5}\right)$$

$$R_y = \Sigma F_y = 0$$

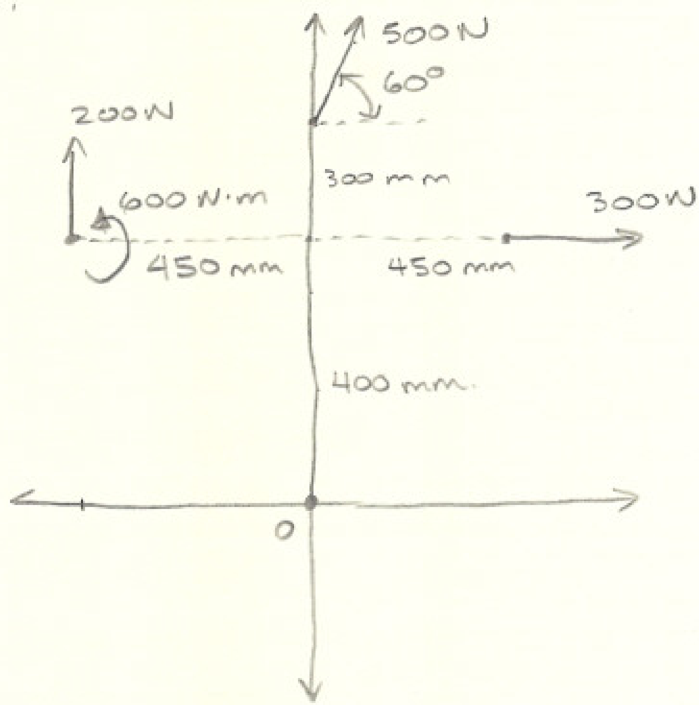
$\therefore \quad R = 0$ (there is no resultant vector)
(this means that this force system is a couple).

b/c $R=0$ we can calculate the couple from any point.

$$\overrightarrow{M_B} = 25 \cdot 3 \circlearrowleft + 125 \cdot \left(\frac{4}{5}\right) \circlearrowright$$

$$= 25 \text{ lb ft} \circlearrowright$$

EX. 4-22.



Determine A. RESULTANT.
B. THE LINE OF ACTION OF THE RESULTANT.
C. THE INTERSECTION OF THE LINE OF ACTION OF THE RESULTANT. $\overline{\omega}$ THE X AXIS.

SOLUTION.

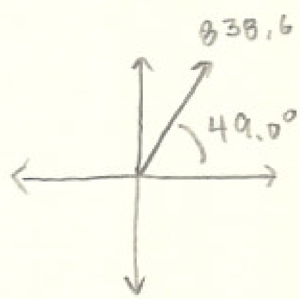
A. $R_x = 0 + 0 + 500\cos 60° + 300 = 550$ N

$R_y = 200 + 0 + 500\sin 60° + 0 = 635$ N

$R = \sqrt{R_x^2 + R_y^2}$

$= 838.6$ N.

$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = 49.0°$

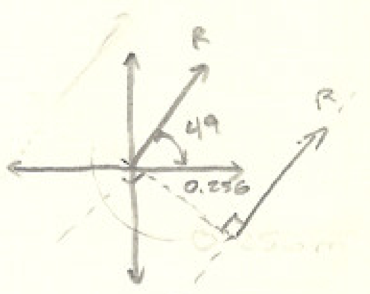


b.

$$d = \frac{\Sigma F_i d_i}{R}$$

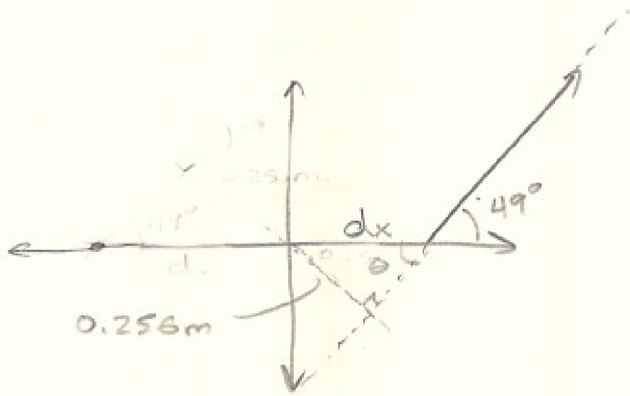
$$= \frac{-(200 \cdot 0.45) + 600 - (500 \cos 60^\circ \cdot 0.7) - (300 \cdot 0.4)}{838.6}$$

$= 0.256$ m.



b/c the $\Sigma F_i d_i$ is positive the resultant is CCW.

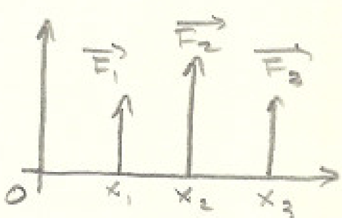
c.



$\theta = 49^\circ$

$$d_x = \frac{0.256}{\sin 49^\circ}$$

§ 4-6.2 NON COPLANAR, PARALLEL FORCE SYSTEMS



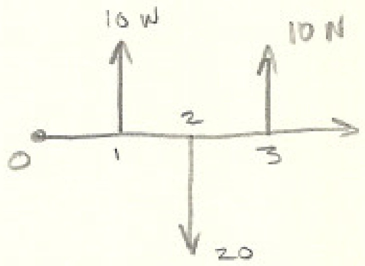
$\vec{R} = \Sigma \vec{F_i}$

$R_x = 0 \quad R_y = \Sigma \vec{F_i}$

$$d = \frac{\Sigma F_i x_i}{R}$$

if R=0 then we have a couple.

EX.



$$R = F_1 x_1 + F_2 x_2 + F_3 x_3$$
$$= 10 \cdot 1 - 20 \cdot 2 + 10 \cdot 3$$
$$= 0$$

$\therefore$ couple

$$M_B = -10 \cdot 1 + 10 \cdot 1 = 0$$

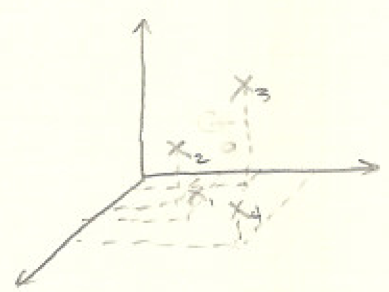
PRACTICAL QUESTIONS.

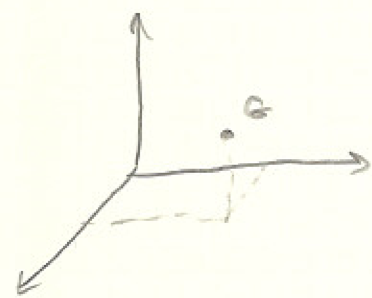
CH 4

6, 7, 21, 36, 62, 66, 85, 87, 98, 102, 113, 117, 118.

CHAPTER 5 DISTRIBUTED FORCES: CENTROIDS AND CENTER OF GRAVITY.

- CENTER OF GRAVITY.





these are equivalent force systems. They have the same resultant vector and moment.

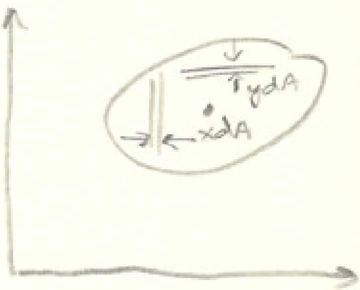
$$\vec{F_G} = \vec{F_{x_1}} + \vec{F_{x_2}} + \vec{F_{x_3}} + \vec{F_{x_4}} = \Sigma \vec{F_{x_i}}$$

$$W = \sum_{i=1}^{n} W_i$$

$$M_x = W d_x = \sum_{i=1}^{n} W_i d_{x_i} \qquad \text{(sub in } y, z\text{)}.$$

CENTROIDS OF COMPOSITE BODIES.

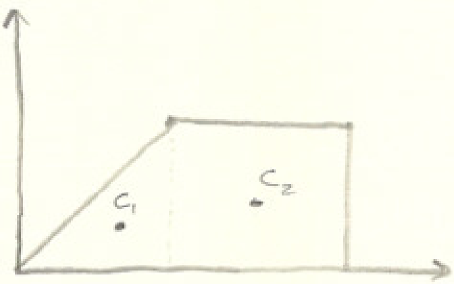
C is the centroid w/ location (x_c, y_c).



$$x_c = \frac{\int x\, dA}{A}.$$

$$y_c = \frac{\int y\, dA}{A}.$$

Ex.



$C_1\ (x_1, y_1)$

$C_2\ (x_2, y_2)$

$$x_c = \frac{\int x\, dA}{A}$$

$$\int x\, dA = \int x_1\, dA_1 + \int x_2\, dA_2$$

$$\int x_1\, dA_1 = A_1 X_1$$

$$\int x_2\, dA_2 = A_2 x_2$$

$$\therefore \int x\, dA = A_1 x_1 + A_2 x_2$$

$$xA = A_1x_1 + A_2x_2$$

$$A = A_1 + A_2$$

$$x_c = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$y_c = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

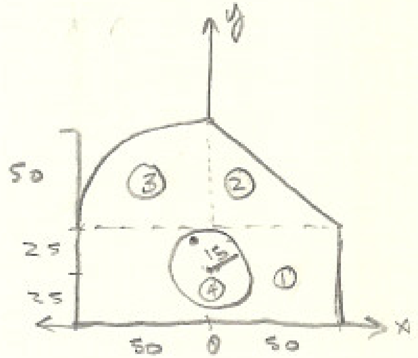
IN GENERAL:

$$x_c = \frac{\Sigma A_i x_{ci}}{\Sigma A_i}$$

$$y_c = \frac{\Sigma A_i y_{ci}}{\Sigma A_i}$$

note that you can use the same idea in dealing with holes.

EX



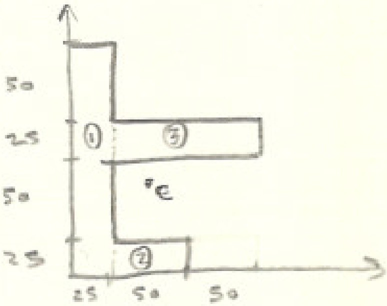
find centroid. in mm.

Solution:

	A_i	x_i	y_i	Ax_i	Ay_i
1	5000	0	25	0	125000
2	1250	16.67	66.67	20838	83338
3	1963	-21.22	71.22	-41605	139805
4	-707	0	25	0	-17075
	7506			-20827	330468

$$x_c = \frac{-20827}{7506} = -2.77\,mm \qquad y_c = \frac{330468}{7506} = 44.0\,mm$$

EX.



	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
1	3750	12.5	75	46875	281250
2	1250	50	12.5	625000	15625
3	2500	75	87.5	187500	218750
	7500			296825	515625

$$x_c = \frac{296825}{7500} = 39.6$$

$$y_c = \frac{515625}{7500} = 68.8$$

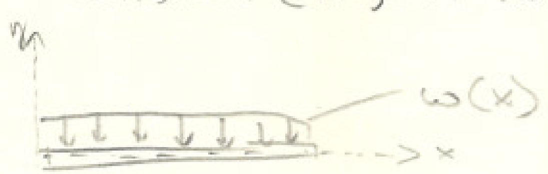
§ 5.6 DISTRIBUTED LOADS ON BEAMS.

A coplanar force system.

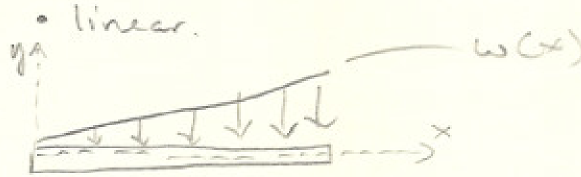
w(x) is the distributed load.

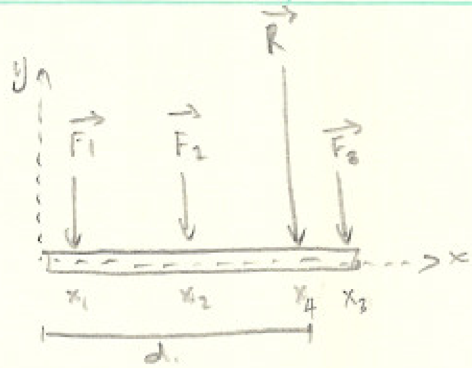
Different kinds of distributed loads.

- constant (uniform load).



- linear.





$$R = \sum F_i = F_1 + F_2 + F_3$$

$$d = \frac{\sum F_i x_i}{R}$$

$$R = \sum w(x)dx = \int_0^L w(x)\,dx$$

$$d = \frac{\sum w(x)dx \cdot x}{R} = \frac{\int_0^L x\,w(x)\,dx}{R.}$$

when $w(x)$ is constant.

$$R = w_0 \int_0^L dx = w_0 L.$$

$$d = \frac{\int_0^L w_0 x\,dx}{w_0 L} = \frac{w_0 \left(\frac{x^2}{2}\right)\Big|_0^L}{w_0 L} = \frac{L}{2}$$

when $w(x)$ is linear. and slope is positive

$$w(x) = w_1 x$$

$$R = \int_0^L w_1 x\,dx = \frac{1}{2} w_1 L^2$$

$$d = \frac{\int_0^L w_1 x(x)\,dx}{\frac{1}{2} w_1 L^2} = \frac{\frac{1}{3} w_1 L^3}{\frac{1}{2} w_1 L^2} = \frac{2}{3} L$$

when the slope is negative.

$$\omega(x) = \omega_1 (-x)$$

$$R = \frac{1}{2} \omega_1 L^2$$

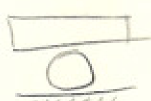
$$d = \frac{1}{3} L$$

CHAPTER 6 EQUILIBRIUM OF RIGID BODIES.

§6.1 FREE BODY DIAGRAMS.

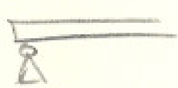
- GRAVITATIONAL FORCE.
- FLEXABLE ROPE.
- SMOOTH SURFACE.
- BALL, ROLLER.

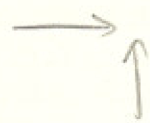
ONE UNKNOWN



- SMOOTH PIN.

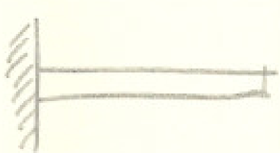
TWO UNIFORMS.



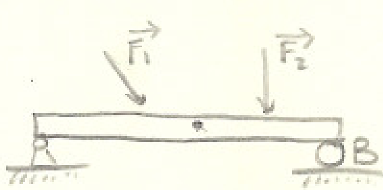


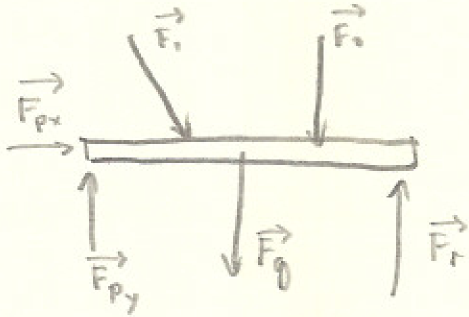
- FIXED BEAM.

THREE UNKNOWNS.



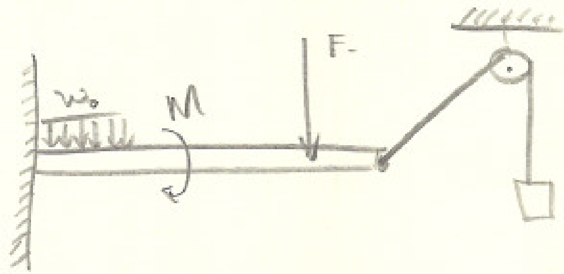
EX. FBD of the Beam.



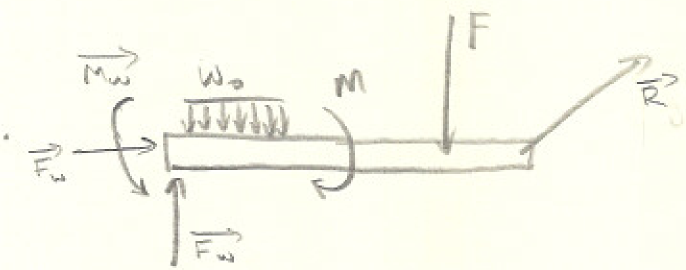


$\vec{F_1}$
$\vec{F_2}$
$\vec{F_{px}}$
$\vec{F_{py}}$
$\vec{F_g}$
$\vec{F_r}$

EX. DRAW FBD.



w_0
M
F



$\vec{M_w}$
w_0
M
F
$\vec{R}$
$\vec{F_w}$
$\vec{F_w}$